OPTIMUM COST DESIGN OF WOOD BEAMS ACCORDING TO EUROCODE5-EC5

CONCEPTION OPTIMALE DES POUTRES EN BOIS SELON EUROCODE5-EC5

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Abstract:

This paper presents the optimal design of wood beams according to Eurocode5 (EC-5). The objective function comprises the cost of the beam and the constraint functions are set to meet design requirements of EC-5. They consist on bending resistance constraints, vertical shear resistance, deflection due to both dead and live loads, design constraints derived from EC-5 and current practices rules. The optimization design process is developed by using Generalized Reduced Gradient (GRG) algorithm. Typically, examples are included to illustrate the applicability of the proposed model. The optimized results are compared with traditional design solutions from conventional design office methods to evaluate the performance of the developed optimum design model. Substantial savings have been achieved through this approach. In addition, the proposed approach is practical, reliable and computationally effective compared to classical designs procedures used by designers and engineers.

Keywords: Optimal Cost Design, Minimum Weight Design, Wood Beams, Eurocode5, Solver.

Résumé :

Cet article présente la conception optimale des poutres en bois selon Eurocode5 (EC-5). La fonction objective comprend le coût de la poutre et les fonctions des contraintes sont définies pour répondre aux exigences de conception de l'Eurocode EC-5. Elles consistent en des contraintes de résistance à la flexion, à la résistance au cisaillement vertical, à la flexion due aux charges permanentes et d'exploitation, aux contraintes de conception issues de l'EC-5 et aux règles en vigueur de pratique courante. Le processus de conception d'optimisation est développé à l'aide de l'algorithme GRG (Gradient Réduit Généralisé). Typiquement, des exemples sont inclus pour illustrer l'applicabilité du modèle proposé. Les résultats optimisés sont comparés aux solutions de conception traditionnelles issues des méthodes classiques du bureau d'études afin d'évaluer les performances du modèle de conception optimal développé. Des économies substantielles ont été réalisées grâce à cette approche. En outre, l'approche proposée est pratique, fiable et efficace par rapport aux procédures de conception classiques
Utilisées par les concepteurs et les ingénieurs.

Mots-clés : Conception optimale, Conception à poids minimal, Poutres en bois, Eurocode5, Solveur.

1. Introduction

Wood is one of the oldest known materials used in construction. Wood construction is growing and continues to evolve in terms of technical regulations, as means of production. It became unavoidable in structural applications. The main advantages of this construction system are prefabricated in workshops and reduction of installation time on site. It is a lightweight material easy to process and repair, also it is widely avail and easy to use. It is sustainable, environmentally friendly; the current ecological issues generate renewed interest in the wood usage in construction. In a difficult environmental context, it is necessary to turn more towards wood construction. A wide range of new structural wood shapes can now be fabricated and used to construct buildings and bridges which have minimal impact on the environment. Wood is particularly attractive since it is renewable and has no carbon footprint when it is harvested in a sustainable way. Timber structures are ecologically sound and comparatively low cost. The material lends itself to innovative design and new types of composites offer reliable, robust and safe materials. Theses following authors have dealt with this topic [1,2, 3, 4, 5, 6]**.**

Wood is an aesthetically pleasing material. It can easily be shaped and connected. It has a very high strength to weight ratio, it is capable of transferring both tension and compression forces and is naturally suitable as a flexural member. It is used for a variety of structural forms such as beam for the industrial construction. As a natural material, wood is unique, innovative, columns, trusses, girders, it is also used in building systems such as piles, deck members, and railway sleepers and formwork for concrete. The shape, form and size to be constructed have being only limited by the manufacturing and the transportation boundaries. Kaveh and Kalatjari [7]have studied the topology optimization of trusses using genetic algorithm, force method and graph theory, Silih et al. [8]have developedthe optimum design of plane timber trusses considering joint flexibility, Silih et al. [9]have presented the shape and discrete sizing optimization of timber trusses by considering of joint flexibility, Yang et al.[10]have analyzed the flexural behavior of wood beams strengthened with hybrid fiber reinforced polymer, Bru et al. [11]have treated the numerical and experimental evaluation of fiber reinforced polymers reinforcement on the mechanical behavior of timber beams, Kaziolasa et al. [12]have published the life cycle analysis and optimization of a timber building and Bru et al. [13]have developed the structural optimization of timber beams with composite materials.

The current development of numerical techniques and the existence of powerful computers provide a solution in order to implement an optimization procedure which is able to find the minimum cost design of wood beams. Although the use of using wood materials is expensive it is an efficient way of getting minimum weight structures, with no reduction in their strength. The economy is achieved by minimization of a cost function. It can create the best structural version by mathematical methods of constrained function minimization, which fulfill the design and fabrication constraints and minimize the cost function. Recently, a number of papers dealing with design on wood have been published by various researchers. The aim of the following paper is to provide another step forward to design and optimize rectangular wood beam using EC-5. Wood beams are generally designed for bending stress and then checked for shear and deflection. The optimization wood structures were performed by the non-linear programming (NLP) approach. In this paper the generalized reduced gradient (GRG) method to solve nonlinear programming is used in order to obtain the minimum cost design and the minimum weight design of wood beams. Various numerical methods have been used in engineering optimization [14-23].

This paper presents the optimal design of wood beams according to Eurocode 5. The objective function comprises the cost of the beam and the constraint functions are set to meet design requirements of Eurocode5. They consist on bending resistance constraints, vertical shear resistance, deflection due to both dead and live loads, design constraints derived from EC-5 and current practices rules. The optimization design process is developed by using Generalized Reduced Gradient (GRG) algorithm. Typically, examples are included to illustrate the applicability of the proposed model. The optimized results are compared with traditional design solutions from conventional design office methods to evaluate the performance of the developed optimum design model. Substantial savings have been achieved through this approach. In addition, the proposed approach is practical, reliable and computationally effective compared to classical designs procedures used by designers and engineers.

2. Formulation of optimization design wood beams

Limit states design for the optimization of wood beams are set in this study in accordance with the current European design code Eurocode5 EC-5[24]. The wood beams having the cross section shown in Figure 1 are considered.

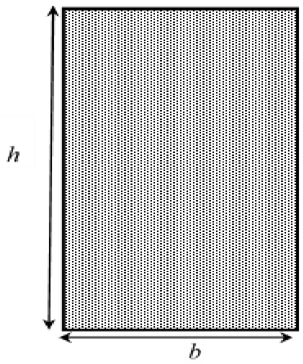

Figure 1: Typical rectangular section of the simple wood beam

Figure 1 : Section rectangulaire type d'une poutre en bois

Wood is composed of fibers that behave better to normal stresses in the longitudinal direction or in the direction of fibers, and poorly to stresses perpendicular to the fibers and to the longitudinal shear stresses. For the short beams, the tangential stress is the required design value, on the other hand for the medium length beams the normal stress is the required design value whereas, for the long beams the beam deflection is the required design value.

2.1. Design variables

The design variables selected for the optimization are listed in Table 1 below.

Table 1 : Definition of design variables

Table 1 : Définition des variables de conception

Design variables	Defined variables
b	Width of the wood beam
h	Depth of the wood beam

2.2. Objective functions

2.2.1. Cost function

The objective function to be minimized in this optimization problem is the total cost of wood beam per unit length of the beam. This function can be defined as:

$$C = \frac{C_t}{L} = C_w bh \qquad (1)$$

Where:

C: Total cost per unit length of wood beam

C_w: Unit cost of wood beam

C_t: Total cost of the element beam

The total absolute cost C_t can then be obtained by using the relation:

$$C_t = CL \qquad (2)$$

Prices include the provision of beams, the fixing system, the techniques used to rise the element in place and includes all works related; such as wood supply, transportation to site, treatment, shaping including assemblies except assemblies used for connecting with the timber already in place, sawing and installation.

2.2.2. Weight function

The weight function to be minimized can be written as follows:

$$W = \frac{W_t}{L} = \rho_w bh \qquad (3)$$

Where: ρ_w is the density of solid wood, W is the unit weight per unit length of the wood beam and W_t the total weight of element wood beam.

$$W_t = WL \quad (4)$$

2.3. Formulation of design constraints

The following constraints for the wood beams are defined in accordance with the design code specifications of the EC-5.

List of the constraints have been considered in this study:

a- Resistance of the wood beam to bending moment:

External moment Resistancemoment and the wood is indoors, where the humidity is controlled.

$$M_{Ed} \Big/ k_{crit} f_{m,k} k_{\text{mod}} k_{sys} k_h bh^2 \Big/ 6\gamma_M \leq 1 \quad (5)$$

b-The buckling of the wood beam doesn't need to be verified when:

$$\sqrt{f_{m,k} \Big/ 0.78 E_{0.05} b^2 \Big/ h(0.9L + 2h)} \leq 0.75 \quad (6)$$

c- Resistance of the composite beam to vertical shear:

External shear Resistance of the structural wood section to vertical shear

$$k_f V_{Ed} / bh \Big/ f_{v,k} k_{\text{mod}} / \gamma_M \leq 1 \quad (7)$$

d- Deflection constraint: the mid-span deflection of simply supported beam under

distribution load (dead load + live load) for the wood beam:

$$\frac{5\left[G(1+k_{def}) + Q(1+k_{def}\psi_2)\right]L^4}{384 E_{0,mean}\, bh^3/12} \leq \delta_{lim} \quad (8)$$

e- Design variables constraints including rules of current practice:

$$b_{min} \leq b \leq b_{\max} \quad (9)$$

$$h_{min} \leq h \leq h_{max} \quad (10)$$

f- Non-negativity variables:

$$b, h > 0 \quad (11)$$

2.4. Formulation of optimum cost design problem of wood beams

The formulation of the optimum cost design of wood beams can be mathematically stated as follows:

For given material properties, loading data and constant parameters, find the design variables defined in Table (1) that minimize the cost function defined in Eq. (1) subjected to the design constraints given in Eq. (5) through Eq. (11).

2.5. Formulation of minimum weight design problem of wood beams

Find the design variables b and h that minimize total weight per unit length defined in Eq. (3), subjected to the design constraints given in Eq. (5) through Eq. (11).

2.6. Solution methodology

The objective function Eq. (1) or the objective function Eq. (3) and the constraints equations, Eq. (5) through Eq. (11), together form a nonlinear optimization problem. The reasons for the nonlinearity of this optimization problem are caused by the expressions of the cross sectional area, bending moment capacity and other constraints equations. Both the objective function and the constraint functions are nonlinear in terms of the design variables. In order to sort out this nonlinear optimization problem, the generalized reduced gradient (GRG) algorithm is used. The Generalized Reduced Gradient method is applied as it has the following advantages:

i) The GRG method is widely recognized as an efficient method for solving a relatively wide class of nonlinear optimization problems.

ii) The program can handle up to 200 constraints, which is suitable for wood beams design optimization problems.

iii) GRG transforms inequality constraints into equality constraints by introducing slack variables. Hence all the constraints are of equality form. The interested reader is directed to [25]for more details of the GRG algorithm.

3. Numerical results

A first typical example problem is now considered, followed by step by step procedure of optimum cost design model then a comparison between the standard design solution and the optimal solution obtained. Finally, a second example is treated in order to illustrate the proposed model of weight and cost minimization for wood beams with EC-5.

Design example A

As previously mentioned, the design constraints are defined in accordance with the code design specifications of EC-5. The optimal solutions are compared with standard design solutions obtained in accordance with EC-5 design code.

The study of a static system corresponds to a wood beam simply supported at its ends and pre-loaded with the uniformly distributed load and designed in accordance with provisions of EC-5 design code as shown in Figure 2.

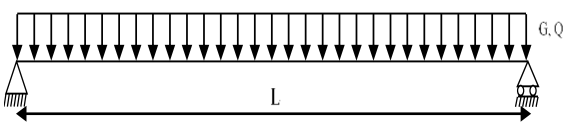

Figure 2. Statically determinate beam with simple supports.

Figure 2. Schéma statique d'une poutre sur appuis simples.

The corresponding pre-assigned parameters are defined as follows:
Input data for loads and wood dimensions:
$L=12.00m$; $b_{min}=0.15m$; $b_{max}=0.25m$; $h_{min}=L/20=0.60m$; $h_{max}=L/12=1.20m$
$M_{Ed}=0.1782MNm$; $V_{Ed}=0.0594MN$; $G=0.004MN/m$; $Q=0.003MN/m$;
$\delta_{lim}=L/200=0.060m$

Input data for wood characteristics:
Strength class of wood C24; solid timber
$f_{m,k}=24MPa$; $f_{m,d}=12.18MPa$; $k_{moy}=0.6$; $\gamma_M=1.3$; $k_{sys}=1.1$, $E_{0.05}=7400MPa$; $E_{0,mean}=11000MPa$; $k_{lef}=0.9$; $k_{crit}=1$; $k_f=1.5$; $f_{v,k}=2.5MPa$, $f_{v,d}=1.15MPa$; $k_{def}=0.80$; $\psi=0.3$; $\rho_w=350kg/m^3$.
Input data for unit costs of construction materials:
$C_w=100€/m^3$

3.2. Step by step procedure of cost optimization model for wood beams

Find the design variables b, h that minimize the total cost of construction material per unit length of composite beam such that:

$$C = 100bh \qquad (12)$$

Subjected to the design constraints:

a- Resistance of the wood beam to bending moment: wood is indoors, where humidity is controlled.
External moment $M_{Ed} \leq$ Resistance moment M_{Rd}

$$\frac{0.1782}{15.84bh^2/7.8} \leq 1 \qquad (13)$$

b-The buckling of the wood beam doesn't need to be verified when:

$$\sqrt{\frac{24}{5772b^2/h(0.9*12+2h)}} \leq 0.75 \qquad (14)$$

c- Resistance of the composite beam to vertical shear:

External shear $V_{Ed} \leq$ resistance of the structural wood section to vertical shear V_{Rd}

$$\frac{0.089/bh}{1.50/1.3} \leq 1 \qquad (15)$$

d- Deflection constraint: the mid-span deflection of simply supported beam under distribution load (dead load + live load) for the wood beam:

$$\frac{5[0.01332]12^4}{4224000bh^3/12} \le 0.06 \qquad (16)$$

e- Design variables constraints including rules of current practice:

$$0.15 \le b \le 0.25 \qquad (17)$$

$$0.60 \le h \le 1.20 \qquad (18)$$

f. Non-negativity variables:

$$b, h > 0 \qquad (19)$$

3.3. Comparison of results between optimal cost design solutions and standard design approach

The vector of design variables from the conventional design solution and the optimal cost design solution using the proposed approach are shown in Table 2 below.

Table 2: Comparison of the classical solution and the optimal solution

Table 2 : Comparaison de la solution classique par rapport à la solution optimale

Design variables	Traditional design	Optimal design with minimum Cost
b[m]	0.25	0.24
h[m]	0.65	0.64
C [€]	16.25	15.36
Gain	/	06%

From the above results, when comparing between the classical and the optimal solutions, you find that a gain equal to 06% can be obtained by using the proposed design formulation.

3.4. Comparison of results between the optimal weight design solutions and the standard design approach

The vector of design variables from the conventional design solution and the optimal cost design solution using the proposed approach are shown in Table 3 below.

Table 3: Comparison of the classical solution and the optimal solution

Table 3 : Comparaison de la solution classique par rapport à la solution optimale

Design variables	Traditional design	Optimal design with minimum Weight
b[m]	0.25	0.24
h[m]	0.65	0.64
W[MN]	0.0005687	0.0005376
Gain	/	06%

From the above results, when comparing between the classical and the optimal solutions, you notice that a gain equal to 06% can be obtained by using the proposed design formulation.

3.5. Comparison of results between the minimum cost design, minimum weight design solutions and the standard design approach

The vector of design variables from the conventional design solution and the optimal cost design solution using the proposed approach are shown in Table 4 below.

Table 4: Comparison of the classical solution and the optimal solution

Table 4 : Comparaison de la solution classique par rapport à la solution optimale

Design variables	Classical solution	Optimal solution with Minimum Cost C	Optimal solution with Minimum Weight W
b[m]	0.25	0.24	0.24
h[m]	0.65	0.64	0.64
Gain	/	06%	06%

3.6. Graphically procedure to obtain the optimum (b_{opt}, h_{opt}, C_{opt})

In this formulation, all the constraints are assumed to be functions of b and h only.

The feasible space can be drawn in the Figure3

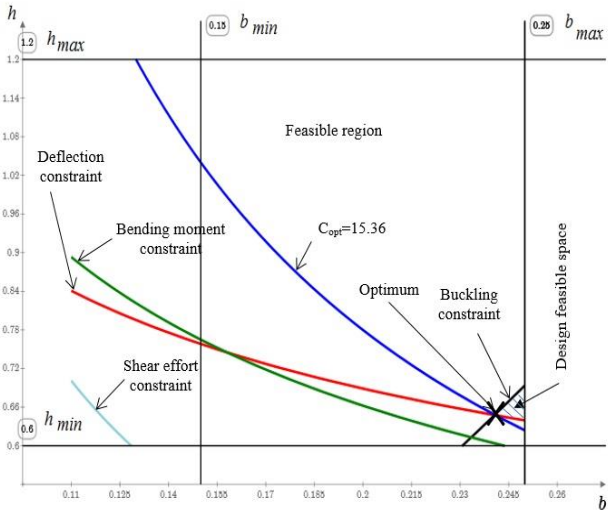

Figure 3: Design space of rectangular wood beam
Figure 3 : Espace de conception d'une poutre rectangulaire en bois

3.7. Design example B

The second design example B corresponds to a wood beam simply supported at its ends and pre-designed in accordance with provisions of EC-5 design code.

The corresponding pre-assigned parameters are defined as follows:

Input data for loads and wood dimensions:

L=10.00m; b_{min}=0.15m; b_{max}=0.25m; h_{min}=L/20=0.50m ;h_{max}=L/12=0.83m

M_{Ed}=0.270MNm; V_{Ed}=0.108MN; G=0.006MN/m; Q=0.009MN/m; δ_{lim}=L/200=0.050m

Input data for wood characteristics:

Strength class of wood C24; solid timber:

$f_{m,k}$=24MPa; $f_{m,d}$=12.18MPa; k_{moy}=0.6; γ_M=1.3; k_{sys}=1.1, $E_{0.05}$=7400MPa; $E_{0,mean}$=11000MPa; k_{lef}=0.9; k_{crit}=1; k_f=1.5; $f_{v,k}$=2.5MPa;$f_{v,d}$=1.15MPa; k_{def}=0.80; ψ=0.3, ρ_w=350kg/m^3.

Input data for unit costs of construction materials:
C_w=100€/m^3

The vector of design variables from the conventional design solution and the optimal cost design solution using the proposed approach are shown in the Table 5 below.

Table 5. Comparison of the classical solution and the optimal solution

Table 5 : Comparison de la solution classique par rapport à la solution optimale

Design variables	Classical solution	Optimal solution Minimum Cost C	Optimal solution Minimum Weight W
b[m]	0.25	0.24	0.24
h[m]	0.80	0.74	0.74
Gain	/	12%	12%

From the above results, when comparing between the classical and the optimal solutions, you can deduce that a significant gain equal to 12% can be obtained by using the proposed design formulation.

4. Conclusions

The following conclusions are drawn from this survey:

- The formulation of the cost design optimization of wood beams can be cast into a nonlinear programming problem, the numerical solution is determined through the use of the Generalized Reduced Gradient algorithm.
- The observations of optimal solutions result reveal that the use of the optimization based on minimum cost and minimum weight design concept may lead to substantial savings in the amount of the construction materials to be used in comparison to classical design solutions of wood beams. These findings showed that optimized cost and weight with GRG algorithm are 06% and 12% economical with respect to traditional design respectively in both examples A and B.
- The objective functions and the constraints considered in this paper are illustrative in nature. This approach based on nonlinear mathematical programming can be easily extended to other sections commonly used in structural design. More sophisticated objectives and considerations can be readily accommodated by suitable modifications of this optimal cost design model.
- The proposed methodology for optimum cost design and minimum weight design are effective and more economical in regards of the classical methods. The results of the analysis show that the optimization process presented herein is effective and its application appears feasible.
- In this study, the optimal values of the design variables are neither affected by the choice of the objective function; nor by the cost function and the weight function. In other terms, for optimal variables there are minimum cost and minimum weight in one shot.

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Nomenclature

The following symbols are used in this paper accordingly to EC-5:

b Width of wood beam;
b_{min} Minimum width of wood beam;
b_{max} Maximum width of wood beam
h Depth of wood beam;
h_{min} Minimum depth of wood beam;
h_{max} Maximum depth of wood beam
L Beam span

I Second moment of area of a section
C Cost per unit length of wood beams

C_w Unit cost of wood

C_t Total cost of wood beams

W Weight per unit length of wood beam

V Volume per unit length of wood beam

G Permanent action

Q Variable action

w Distribution load (dead load + live load)

γ_G Partial factor for permanent actions

γ_Q Partial factor for variable actions

γ_{M} Partial factor for material properties

ψ_2 Factor for the quasi-permanent value of a variable action

δ_{lim} Vertical deflection limit of wood beams

M_{Ed} Maximum design bending moment

M_{Rd} Maximum resistance moment of the wood

V_{Ed} Maximum design shear force

$V_{,Rd}$ Maximum resistance of the structural

wood section to vertical shear

C24 Class of wood

$E_{0.05}$ Fifth percentile (characteristic value) of modulus of elasticity

$E_{0,\mathrm{mean}}$ Mean value of modulus of elasticity parallel to the grain

$f_{\mathrm{m,k}}$ Characteristic bending strength

$f_{\mathrm{m},d}$ Design bending strength

$f_{\mathrm{v,k}}$ Characteristic shear strength

$f_{\mathrm{v,d}}$ Design shear strength

k_{crit} Factor used for lateral buckling

k_{ef} Exponent factor to derive the effective number of fasteners in a row

k_{def} Deformation factor

k_{h} Depth factor

k_{mod} Modification factor for duration of load and moisture content

k_{sys} System strength factor

λ_{rel} Relative slenderness ratio corresponding to bending

ρ_{w} Density of solid wood

$\sigma_{\mathrm{m,crit}}$ Critical bending stress