BEHAVIOR SOIL SIMULATION OF SHALLOW FOUNDATION

SIMULATION DU COMPORTEMENT DE SOL D'UNE FONDATION

Réception : 18/03/2020 **Acceptation : 09/05/2020** **Publication : 25/06/2020**

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Abstract

This paper presents a finite element approach to analyze the response of shallow foundations on soils and the time-dependent settlement is solved. Karl von Terzaghi was the first to present a comprehensive theory for the evaluation of the ultimate bearing capacity of rough shallow foundations. This theory states that a foundation is shallow if its depth is less than or equal to its width. Later investigations, however, have suggested that foundations with a depth, measured from the ground surface, equal to 3 to 4 times their width may be defined as shallow foundations. Terzaghi developed a method for determining bearing capacity for the general shear failurecase in 1943. The equations are given below. The prediction of collapse loads under steady plastic flowconditions is one that can be difficult for a numerical model to simulate accurately. A simple example of a problem involving steady-flow is the determination of the bearing capacity of a footing on an elastic-plastic soil. The mathematical model was analyzed by the Finite Element Method using the ANSYS® software system.

Keywords. Terzaghi, Foundation, ANSYS®, Finite Element Method (FEM)

1- Introduction

In the current applications, the bearing capacity of shallow foundations is usually evaluated using the well-known equation proposed by Terzaghi [1]

$$q_{\lim} = qN_q + C'N_c + \frac{1}{2}\gamma BN_\gamma \qquad (1)$$

Where:

- $q_{\lim}$: is the bearing capacity pressure,
- q : is the uniformly distributed surcharge replacing the over burden soil at the level of the foundation base,
- C': is the cohesion intercept of the soil, B is the foundation width,
- γ : is the soil unit weight, Nq, Nc and N_γ are the bearing capacity factors which depend on the soil shearing resistance angle φ'.

The mechanical behavior of soils was coded only after the introduction of the "concept of effective stresses" [1] which marked the birth of Soil Mechanics starting from the general structure of Continuum Mechanics from which it derives**.**

Equation one refers to strip footings resting on homogeneous and dry soil with centrally located vertical load and symmetrical failure pattern. To extend Terzaghi's solution to more general conditions than those above specified, a great number of theoretical studies were conducted in which numerical techniques were used to accountfor reliably the effects of important factors on the bearing capacity calculation, such as footing shape, roughness of base, inclination and eccentricity of loading, ground surface inclination, groundwater **[2]**.

Karl von Terzaghi was the first to present a comprehensive theory for the evaluation of the ultimate bearing capacity of rough shallow foundations. This theory states that a foundation is shallow if its depth is less than or equal to its width **[3]**. Later investigations, however, have suggested that foundations with a depth, measured from the ground surface, equal to 3 to 4 times their width may be defined as shallow foundations. Terzaghi developed a method for determining bearing capacity for the general shear failure case in 1943. The existence of the exact solution to Terzaghi's equation emerged from researches regarding the description of geotechnical phenomena through direct or inverse hyperbolic laws[4].

This concept is based on the inner structure of soils that are composed of solid skeleton and inter-particle gaps. These pores are more or less inter-connected and through them run fluids of different nature. Therefore, in view of a necessary simplification of the mathematics of associated phenomena, the concept of effective stresses requires the soils to be assimilated to bi-phasic systems composed of a solid skeleton saturated with water, *i.e.* two continuous means that act in parallel and share the stress status:

$$\sigma'_{ij} = \sigma_{ij} - u_0\delta_{ij} \qquad (2)$$

In Equation 2 there is the tensor of the total stresses exerted by the solid skeleton (σ_{ii}) the hydrostatic pressure exerted by the fluid (u_0, known as interstitial pressure) and Kronecker's delta (δ_{ii}); furthermore, from a merely phenomenological point of view, Eq. 2 attributes the soil shear resistance only to effective stress, independent of the presence of the fluid. At all points structural, the problem focus is on the permeability coefficient ($k = ms^{-1}$) Eq. 3 that, by expressing the capacity of a soil to transmit a fluid, takes on the character of a velocity and varies approximately in the range.

$$k = \frac{C_v\gamma_w a_v}{1+e_0} \qquad (3)$$

Where:

- a_v : Compressibility coefficient
- C_v: Factor of consolidation (m^2s^{-1})
- γ_w: unit weight of water [kN/m^3]

Considering the above, the one-dimensional consolidation equation [1,3] describes the hydraulic behavior of soils in transient conditions by making it possible to simulate the variation in time of interstitial overpressures (U_e), generated -for example-by the load induced by a foundation or by a road embankment (Fig.1), with consequent visco-elastic settlements to which corresponds a structural reorganization of the solid skeleton, with reduction of porosity and, concurrently, of the degrees of freedom.

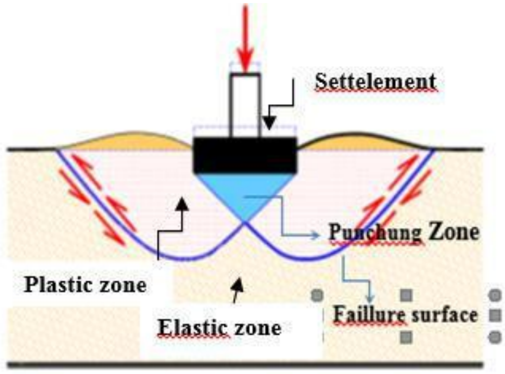

Figure 1: The load transmitted by a foundation always causes interstitial overpressures whose dissipation depends on soil permeability that in turn depends on porosity; consolidation may last from some minutes (loose sands) to tens of years (very stiff clays).

This figure illustrates three zones as:

- Zone 1st said punching zone located under the foundation. The soil grains are relatively confined.
- In the 2nd Zone said plastic zone, the soil grains tend to displace laterally at the edges from under the load.
- In the 3rd zone the soil grains are relatively in stat equilibrium. It's said elastic zone

In reality, in these materials it occurs that some portions of the soil first fail owing to loading, with the shear strain that is located in a zone of limited thickness (shear surface). Owing to the consequent redistribution of stress, the shear band propagates in the soil and a slip surface progressively develops up to causing the collapse of the soil-foundation system.

It should be noticed that Eq. 4 is analogous to Fourier's law on heat propagation to the point that you can define the theory of consolidation as the simulation of the propagation of stress-induced interstitial pressures in the subsoil. This conditional transaction results from the same physical as the differential diffusion equation. Differential equation of transient heat conduction in an isotropic body [5].

$$\frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) = \frac{\partial T}{\partial t} \quad (4)$$

Differential equation of primary consolidation in an isotropic body[6]:

$$\frac{\partial}{\partial z}\left(C_v \frac{\partial u}{\partial z}\right) = \frac{\partial u}{\partial t} \quad (5)$$

The primary consolidation could be characterized as a hydraulic flow in a porous media. The premises and substitutions in the solution are:

- T : Temperature ⇒ u = pore pressure

- λ : Coefficient of thermal conductivity

- C_V: Coefficient of consolidation

With the boundary condition u = 0 on z = 0 and the initial condition $U = U_0$ at t = 0, equation (5) can be solved

In cylindrical coordinates, [8] exposed the equation below:

$$c_v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \frac{\partial u}{\partial t} \quad (6)$$

2- Elasto-viscoplastic model

In most of these studies, it was assumed that failure occurs simultaneously along the slip surfaces that develop in the soil mass beneath the footing. However, this failure process is progressive in nature owing to the fact that the plasticstrains induced in the soil by loading are markedly non-uniform. Asa consequence, the soil shear strength is not simultaneously mobilized at all points of a slip surface. It was also assumed that the soil strength parameters remain unchanged even if large strains are inducedby loading. This assumption is inadequate for soils that are characterized by a pronounced strain-softening behavior, such asdense sands. In reality, in these materials it occurs that some portions of the soil first fail owing to loading, with the shear strain thatis located in a zone of limited thickness (shear band). With increasing strain within this zone, soil strength reduces from peak towards the critical state. Owing to the consequent redistribution of stress, the shear band propagates in the soil and a slip surface progressively develops up to causing the collapse of the soil-foundation system. At failure, the average strength mobilized along the slip surfaceis generally less than the peak strength and greater than the strength at the critical state of the sand under consideration.

Under the assumption of small strains, the total strain rate tensor for an elasto-viscoplastic material can be written as follows:

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{vp}$$

Where ε_{ij}"is the total strain rate tensor, ε_{ij}^{e} and ε_{ij}^{vp} are the elastic and the viscoplastic strain components, respectively. The tensor ε_{ij}^{e} is defined as:

$$\varepsilon_{ij}^{e} = C_{ijhk}\sigma'_{hk}$$

Where σ'_{hk} is the effective stress rate tensor, and C_{ijhk} is the elastic compliance tensor.

The occurrence of a progressive failure in loading tests and centrifugal tests concerning footings on granular soils, was observed by several authors [7–8]. On the basis of these evidences, Perkins and Madson [9] proposed a semi-empirical procedure to estimate the bearing capacity of footings on dense sands. However, a progressive failure process can be success fully predicted using an approach that accounts for properly the strain-softening behavior of the soil and it is able to simulate reliably the formation and development of shear zones within the soil. Finite element approaches with these characteristics were employed by Siddiquee et al. [10] and Banimahd and Woodward [11] to analyze the response of footings on granular soils to loading. Generalised elasto-plastic strain-softening models are incorporated in these approaches for modelling the behavior of the soil involved. However, these models generally require that a significant number of specific constitutive parameters is determined.

In this study, a finite element approach is proposed to analyze the response of shallow foundations resting on soils withstrain-softening behavior. This approach utilizes a non-local elasto-viscoplastic constitutive model in conjunction with a Mohr–Coulomb yield function in which the strength parameters are reduced with the accumulated deviatory plastic strain. The proposed method requires few material parameters as input data. In addition, most of these parameters can be readily obtained from conventional geotechnical tests. The method is first applied to simulate the experimental results from some physical model tests concerning a rigid strip footing resting on a layer of dense sand in which a thin layer of weak material is located at different depths from the ground surface. Then, the progressive failure process that occurs in homogeneous sand owing to loading is analyzed; and the main aspects of this process are discussed.

3- Analysis of physical model tests

Muscolino [12] carried out a series of physical model tests under1g plane strainin conditions using a rigid footing placed on a dry sand layer in which a thin layer of weaker material was located at different depths from the surface. The main objective of Muscolino's study was to analyze the influence of this thin layer on the bearing capacity of shallow foundations. Further experimental results from these model tests can be found in Valore et al. [13]. As cheme of the tests is shown in Fig. 2 in which the dimensions of the soil layer and those of the footing are indicated.

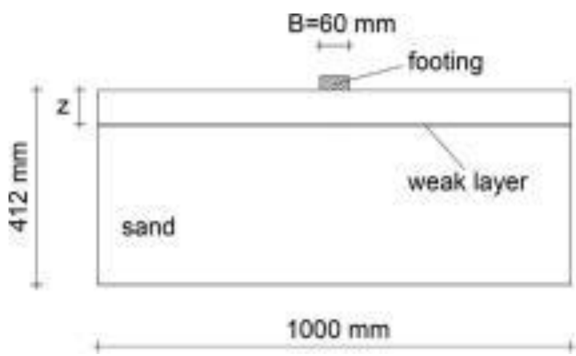

Figure 2: Scheme of the model tests performed by Muscolino [12].

The sand was deposited in a box of plexiglas reinforced by steel frame. The inner sides of this box were covered by a glass sheet smeared with oil to minimize the effect of friction at the interface between the soil and the box walls. The weak layer was 3 mm thick and consisted of wet bentonite or dry talcum. A uniformly distributed load p = 2.9 kPa was initially applied to level the layer surface. This loadwas then removed before placing the footing which was constitutedby a block of aluminium with rectangular cross-section.

4- Hydro – Mechanical coupling

The mechanical interaction between groundwater and the porous geologic media it permeates is central to the phenomenon of groundwater flow [14]. Calculations which take this interaction into account are called hydro-mechanically (HM) coupled. Different types of mechanical behavior are of relevance. In the subsurface, typically described as porous media, often only elastic dilatation is taken into account, not fracture. Although no materials are actually linearly elastic over a wide range of stresses, elastic constitutive models are mostly sufficiently accurate for rock mechanics [15]. The theory describing the elastic behavior of porous media is called poroelasticity. Two basic phenomena underlie poro elastic behavior as introduced in the theory of poro-elasticity [16].

- Solid-to-fluid coupling occurs when a change in applied stress produces a change in fluid pressure or fluid mass.
- Fluid-to-solid coupling occurs when a change in fluid pressure or fluid mass produces a change in the volume of the porous material.

The theory consequently addresses the transient coupling between the deformation of rock and fluid flow within the rock. Different mathematical approaches are available; see [17], [18], [19].

In the case of hydro-mechanical coupling it requires the solution of the groundwater flow (Darcy equation) and the mechanical behavior described for instance following. For thermo-coupling additionally the heat - and mass-transport equations have to be coupled dynamically.

4.1- Coupling Methods

Coupled-field analyses are useful for solving problems where the coupled interaction of phenomena from various disciplines of physical science is significant.

Several examples of this include : an electric field interacting with a magnetic field, a magnetic field producing structural forces, a temperature field influencing fluid flow, a temperature field giving rise to thermal strains and the usual influence of temperature-dependent material properties. The latter two examples can be modeled with most non-coupled-field elements, as well as with coupled-field elements [20]. There are basically two methods of coupling which are distinguished by the formulation of finite element techniques used to develop the matrix equations. These are illustrated here with two types of degrees of freedom ({*x1*}, {*x2*}). Most physical tasks deal with one way coupling, which means, for example, the effect of the thermal field will cause deformations (displacements, rotations); however, displacements do not cause modifications of a thermal field.

1. Strong (also matrix, simultaneous, or full) coupling - where the matrix equation is of the form:

$$\begin{bmatrix} [K_{11}][K_{12}] \\ [K_{21}][K_{22}] \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \quad (7)$$

The coupled effect is accounted for by the presence of the off-diagonal sub matrices $[K_{12}]$ and $[K_{21}]$. This method provides for a coupled response in the solution after one iteration.

2. Weak (also load vector or sequential) coupling - where the coupling in the matrix equation is shown in the most general form:

$$\begin{bmatrix} [K_{11}(\{X_1\},\{X_2\}] & 0 \\ 0 & [K_{11}(\{X_1\},\{X_2\}] \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} F_1\{X_1\},\ \{X_2\} \\ F_2\{X_1\},\ \{X_2\} \end{Bmatrix} \quad (8)$$

The coupled effect is accounted for in the dependency of [K11] and {F1} on {X2} as well as [K22] and {F2} on {X1}. At least two iterations are required to achieve a coupled response. The coupled pore-pressure and the program models porous media containing fluid by treating the porous media as a multiphase material and applying an extended version of Biot consolidation theory. The flow is considered to be a single-phase fluid. The porous media is assumed to be fully saturated.

5- Finite element analyses model with ANSYS software

The applications deriving from eq. (8) required an initial phase of in-depth study leading, at the moment, to two different uses, aimed at analyzing the temporal evolution of the consolidation phenomenon. Take for example a 20 m thick clay bank resting on gravels, that corresponds to a drainage path with a length H = 10 meters; on this bank a static load N = 100 Pa is applied, whereas the performance of oedometer tests showed that Cv = 0.02 m2/day. Furthermore, it is assumed that load application times are negligible as against clay filtration times, thus leading to a system that initially is undrained and capable of developing a u = N = 100 Pa in relation to the incompressibility of both solid skeleton and pore fluid. The soil and the consolidation characteristics are showing in the following table:

Table 1: Soil characteristics, Biot and Consolidation Coefficients

Material Properties	Loading
E=1000Pa Nu=0.3 W=1m k=0.267e-4m/s t=374530mn c=1 Cv=0.02m^2/day	N=100Pa W=1m

Where:

- E : Young's Modulus
- Nu : Poisson's Ration
- k : Soil Permeability
- t : time
- c : Biot Coefficient
- N : Static load
- Cv : Consolidation Coefficient

Figure 3 below illustrate the Ansys model boundary conditions.

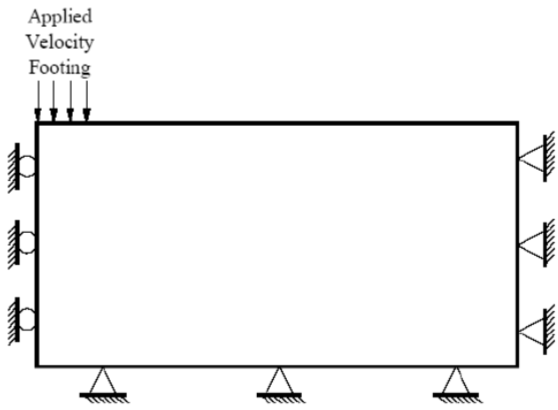

Figure 3: Ansys model boundary conditions

5.1- Structure discretization

The Finite Element Analysis (FEA) method is a powerful computational technique for approximate solutions. ANSYS is engineering software, worldwide used by researchers for simulation. It develops general purpose of finite element analysis. To create the finite element model in ANSYS there are multiple tasks that have to be completed for the model to run properly. Models can be created using command prompt line input or the Graphical User Interface (GUI). For this model, the GUI was utilized to create the model. The following figure (Fig 4) shown the two dimensional FEM discretization.

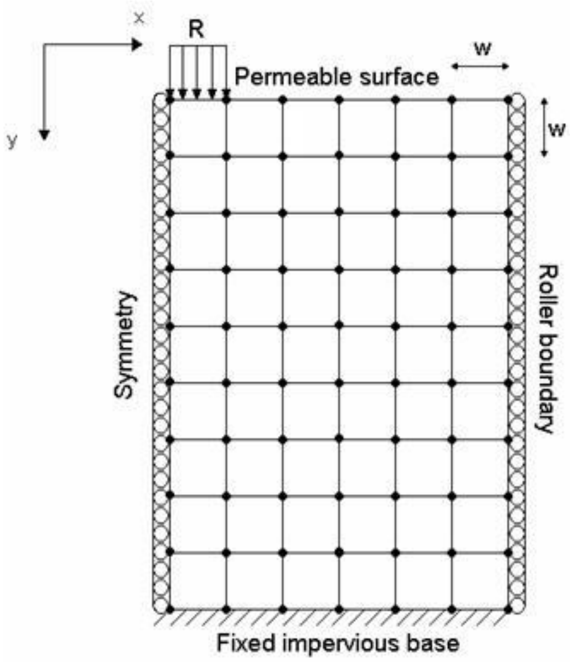

Figure 4: Two-Dimensional Finite Element discretization Model

Not that the commercial software ANSYS, is a powerful structural analysis simulation based on the finite element method. It is thus used in the analysis and evaluation of the transient thermal transfers in the materials, the calculation of the residual stress field during loading, etc. Analysis by ANSYS includes two steps, namely a step of modeling and a calculation step. The first step is to model the finite element structure by choosing an appropriate item to the type of analysis to be performed, such as for example: The element **CPT213** is coupled Pore-Pressure Mechanical Solid. CPT213 (Fig. 5) can be used as a plane strain or axisymmetric element. The element has stress stiffening, large deflection, and large strain capabilities. Various printout options are also available. The element has quadratic displacement behavior and is well suited to modeling curved boundaries. The following figure (6) showed the structural response under the quadratic load.

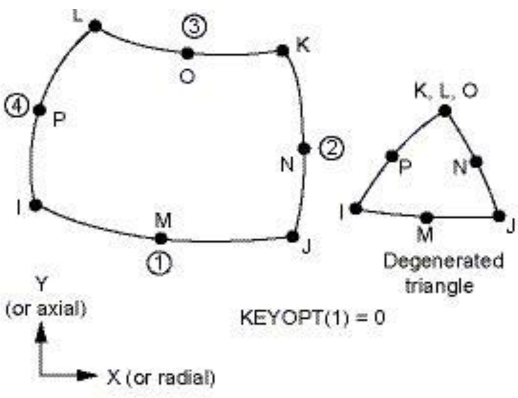

Figure 5: Solid CPT 213 Geometry [21]

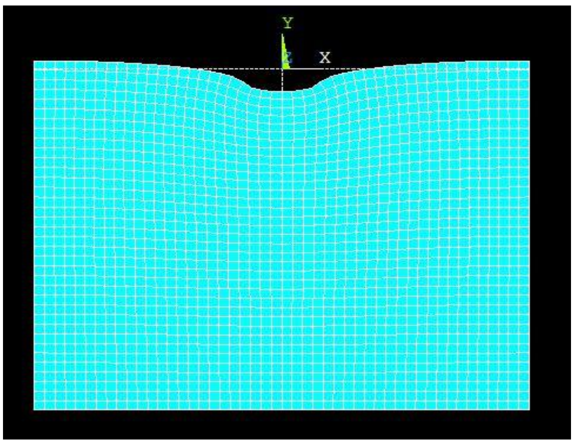

Figure 6 : Structural deformations under load

5.2-Numerical results and Discussions

The behavior of uniformly loaded sub-soil can be studied on a numerical model, see Fig.5 using the ANSYS software. The sub-soil investigated was solved in a plane strain state, modeled from 8 nodes CPT213elements. The region investigated with dimensions of 10 x 20 m was divided into elements of the sizes 1 x1 m (See Fig. 4). We achieved the symmetry of the model by inputting the boundary support conditions. The interaction between the foundation structure and the subgrade was modeled using PLANE182. PLANE182 is used for 2D modeling of solid structures. The element can be used as either a plane element (plane stress, plane strain or generalized plane strain) or an axisymetric element. The material characteristics of the subsoil were defined accordingto this particular model.The linear elastic model of the homogeneous isotropic material is defined above (Sect 5). Additional characteristics defining the primary consolidation that were considered in the calculations are the consolidation factor, permeability and the coefficient of the transaction that resulted from the experimental tests.The relation of the final values of the deformations in particular load levels can be presented graphically as a soil compressibility curve, which is used for the principal expression of soil deformation properties.

Series of computations have been carried out using ANSYS14 software to study the settlement behavior of soil under shallow foundation are chowed in the following figure, were we will illustrate, an contour plot of pore pressure in sub-soil, Von Mises, elasto–viscoplastic strain, elasto– visco plastic stress , shear curve in the plane XY. The settlement analysis was made using the method which has been described above; figure 7 below shown the contour plot of Elasto– viscoplastic deformations “ε_y”.

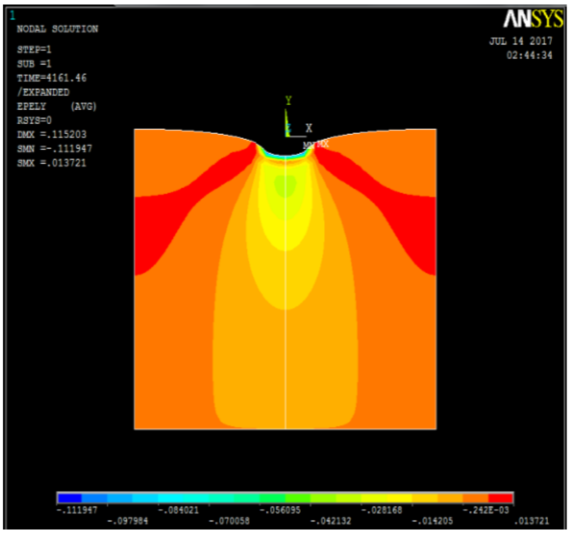

Figure 7: Contour plot of elasto – visco plastic "ε_y "soil strains

Figure 8 shown a curve shear soil at the first time (t = 100mn).

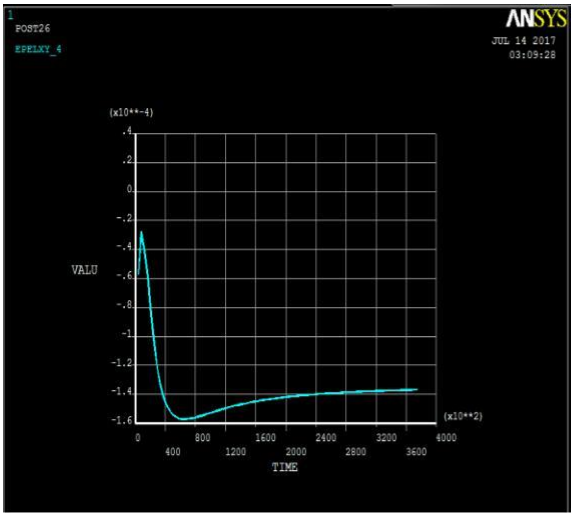

Figure 8: Illustration of shear curve XY Plane

The following figure 9 shown the pore pressure distribution in the sub-soil.A notice able decrease of pore pressure is recognized. This change follows the theory of primary consolidation as the cause of the creep of the subsoil, which means additional deformations.

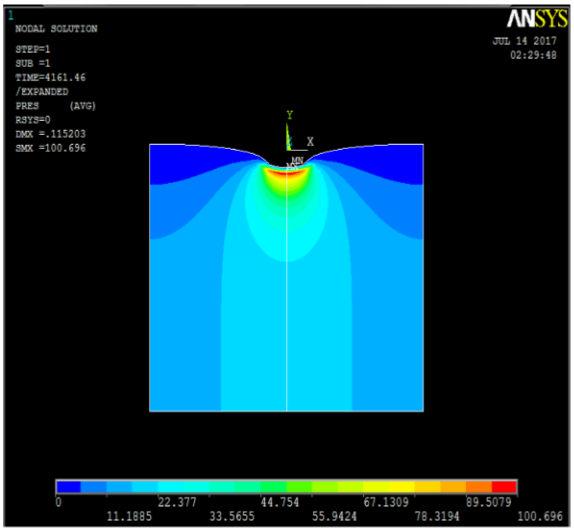

Figure 9 : Contour plot of pore pressure in sub-soil

Figure 10, Figure 11 and Figure 12 illustrated the evolution of the "σ_x , σ_y" stresses. A notice that the stresses evolutions are symmetric and we notes that they are decrease over the time t_0 to the end time t_2.

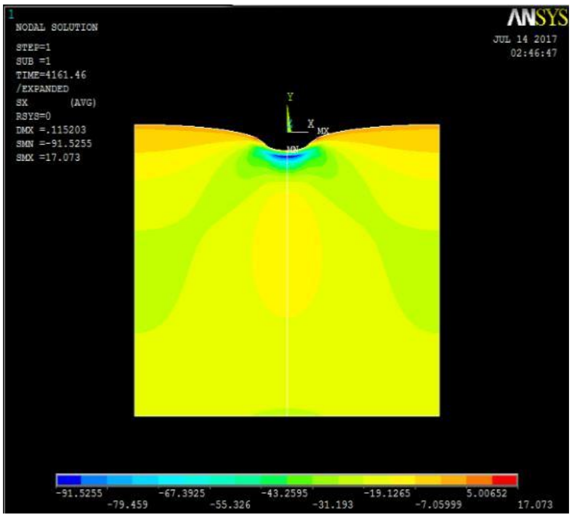

Figure 10: Contour plot of the σ_x stresses

Figure 13 and Figure 14 shown the contour plot of Von Mises elasto–viscoplastic and the curve of Von Mises strain at start time t_0 to the end time t_2 of the investigation. It is noted that the evolution of the constraints is carried out in two phases.

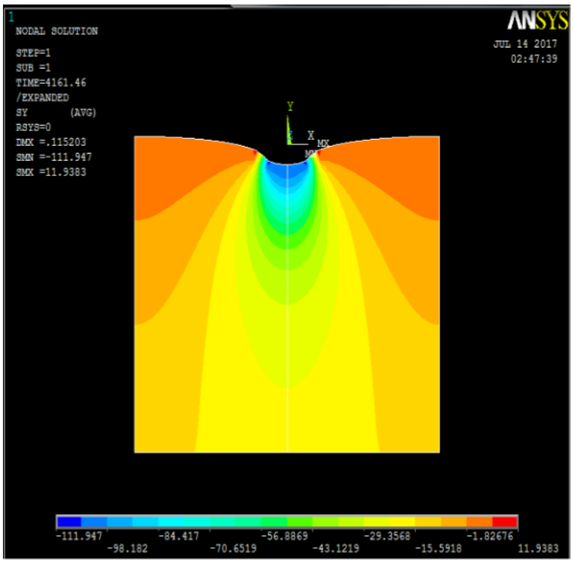

Figure 11: Contour plot of the σ_y stresses

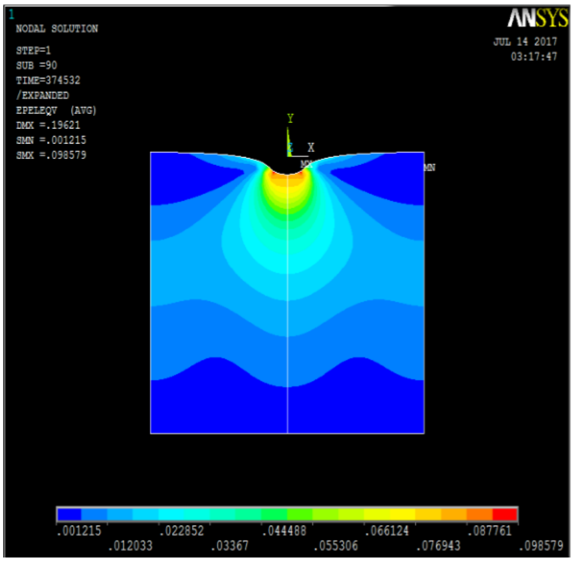

Figure 13: Contour plot of Von Mises elasto – visco plastic

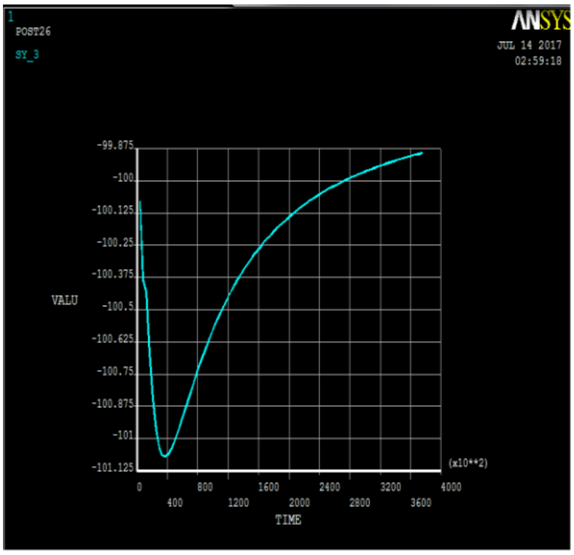

Figure 12: illustration curve of the σ_y stresses

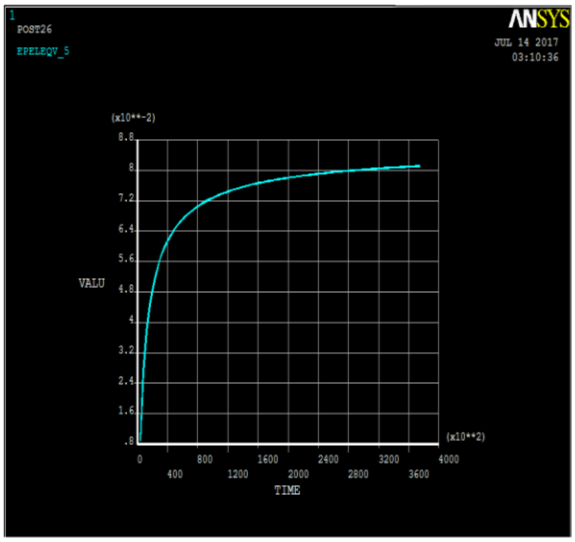

Figure 14: Illustration of Von Mises elasto - visco-plastic strain curve

6-Conclusion

In this paper the possibility of the simulation of the real time behavior of settlement sub-soil under shallow foundation by using the finite element method was presented. Raphson solution, however, the solution converged to the results of the experimental measurement very well.

A fairly good agreement has been found between the theoretical results achieved using the present approach with the experimental results from some physical model tests concerning a rigid andrough strip footing on a dense sand layer in which a thin inclusion of weaker material is located at different depths from the layer surface.

The case of a rough strip footing on a homogeneous sand layer has been also analyzed. The associated settlement curve experiences a pronounced peak that drastically reduces to the ultimate value. In addition, the results have clearly shown the nature progressive of the failure process that occurs in the soil with a Prandtl-type mechanism. In particular, a wedge-shaped zone and two radial zones first develop in the soil beneath the loaded area. These zones are delimited by shear bands with very high values of the accumulated deviatory plastic strain. Then, significant plastic strains are also induced in the outer zones of the soil on the sides of the footing. However, these strains are much lower thanthose generated in the former zones of the soil.

Significant results were obtained concerning the vertical deformations over time. The time-dependent settlement of the foundations of buildings can be calculated with the proper application of this FEM model to geological subsoil conditions. The results from the set up models showed a decrease in the required time for primary consolidation as wellas reducing the deformation zone and total settlementwas introduced.

Bibliographie

[1] Terzaghi K., Theoretical soil mechanics. New York: Wiley; 1943.

[2] E. Conte et al., “Progressive failure [2] E. Conte et al., “Progressive failure with strain-softening behavior Computers and Geotechnics 54 (2013) p. 117–124.

[3] Terzaghi K. and Fröhlich O.K., Theorie der Setzung von Tonschichte, Leipzig/Wien Deuticke (1936).

[4] Di Francesco R. Leggi di degradazionedei geomateriali. In Giornate di Studio dell Associazione Italiana di Difesadel Suolo, Fossa, L’Aquila – Italy (2008).

[5] Zienkiewicz, O.C., The Finite Element Method, McGraw – Hill, London, 551 pp., (1977)

[6] Terzaghi, K., Theoretical Soil Mechanics, John Wiley & Sons, Ltd, New York, pp. 311, (1943)

[7] Yamaguchi H, Kimura T, Fujii N., On the influence of progressive failure on the bearing capacity of shallow foundation in dense sand. Soils Found, 1976; 16(4):11–22.

[8] Aiban S.A., Znidarcˇic´ D., Centrifugal modelling of bearing capacity of shallow foundations on sand. J. Geotech. Eng., ASCE, 1995; 121(10):704–12.

[9] Perkins S.W., Madson C.R., Bearing capacity of shallow foundations on sand : a relative density approach., J. Geotech. Geoenviron. Eng., ASCE, 2000; 126(6):521–30.

[10] Siddiquee M.S.A., Tanaka T., Tatsuoka F., Tani K., Morimoto T., Numerical simulation of bearing capacity characteristics of strip footing on sand. Soils Found, 1999; 39(4):93–109.

[11] Banimahd M., Woodward P.K., Load-displacement and bearing capacity of foundations on granular soils using a multi-surface kinematic constitutive soil model. Int. J. Numer. Anal. Meth. Geomech., 2006; 30:865–86.

[12] Muscolino S.R., L'influenza delle interfacce e dei dettagli costitutivi minori sulla stabilita delle fondazioni e delle gallerie. PhD thesis, University of Palermo; 2001.

[13] Valor C., Muscolino S.R., Ziccarelli M., L'influenza dei dettagli di costituzione del terrenosulla stabilita delle fondazioni superficiali''. In: Proc V Convegno Nazionale dei Ricercatori di Ingegneria Geotecnica, 2006, Bari, vol. 1; 2006. p.231–57.

[14] Ingebritsen, S.E., Sanford, W.E. & Neuzil, C.E. Groundwater in Geologic Processes, 2nd Edition, DOI: 10.2277/ 0521603218, 564 pages, (2006).

[15] Kolditz, O., Görke, U-J., Shao, H., & Wang, W., Thermo-Hydro-Mechanical-Chemical Processes in Porous Media: Benchmarks and Examples. Heidelberg: Springer, (2012)

[16] Wang, H.F., Theory of Linear Poro-elasticity – with Applications to Geomechanics and Hydrogeology. Princeton University Press, 287 pages, (2000)

[17] Verruijt, A., Elastic storage of aquifers, in De Wiest, R.J.M., (ed.), Flow Through Porous Media: New York, Academic Press, p. 331–376, (1969)

[18] Smith, I.M. & Griffiths, D.V., Programming the finite element method, 2nd ed. Chinchester, John Wiley and Sons, pages 469, (1988)

[19] Leake, S.A., & Hsieh, P.A., Simulation of deformation of sediments from decline of ground-water levels in an aquifer underlain by a bedrock step, in Prince, K. R., and Leake, S.A., eds., U.S. Geological Survey Subsidence Interest Group conference - proceedings of the technical meeting, Las Vegas, Nevada, February 14-16, 1995: U. S. Geological Survey Open-File Report, (1997)

[20] Kohnke, P., *ANSYS*, User´s Manual for Revision 8.1, Vol. I-IV, Swanson Analysis System, Inc., (2005)

[21] ANSYS Element Reference, General Element Features, CPT213 Element Description