MINIMUM COST DESIGN OF REINFORCED CONCRETE RECTANGULAR SECTION IN BENDING UNDER ULTIMATE LOADS

CONCEPTION A COUT MINIMAL DE SECTION RECTANGULAIRE EN BETON ARME EN FLEXION SOUS CHARGEMENT ULTIME

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Abstract - The present paper deals with the structural design optimization of reinforced concrete rectangular sections under ultimate design loads. An analytical approach of the problem, based on a minimum cost design criterion and a reduced number of design variables, is developed. It is shown, among other things, that the problem formulation can be cast into a non linear mathematical programming format. A typical example is presented to illustrate the applicability of the formulation in accordance with the current French design code BAEL-99. The results are then confronted to those derived from current design practice. The optimal solution shows clearly that significant savings can be made in the predicted absolute costs of the construction materials to be used.

Keywords: non-linear optimization, reinforced concrete rectangular beam, ultimate design loads, BAEL-99, algorithm.

Résumé - Le présent article traite de l'optimisation de la conception structurale des sections rectangulaires en béton armé sous des charges de conception ultimes. Une approche analytique du problème, basée sur un critère de conception de coût minimal et un nombre réduit de variables de conception, est développée. Il est montré, entre autres, que la formulation du problème peut être inscrite dans un format de programmation mathématique non linéaire. Un exemple typique est présenté pour illustrer l'applicabilité de la formulation selon le code de conception français actuel BAEL-99. Les résultats sont ensuite confrontés à ceux obtenus de la pratique de conception actuelle. La solution optimale montre clairement que des économies significatives peuvent être réalisées dans les coûts absolus des matériaux de construction mis en œuvre.

Mots clés: optimisation non-linéaire, poutre rectangulaire en béton armé, chargement ultime de conception, BAEL-99, algorithme.

Introduction

Rectangular beams are the most frequently used elements in industrial building frames in reinforced concrete construction. They are typically used to support building floors in reinforced concrete construction. For repeated use of such elements, as in the case of prefabrication in reinforced construction, a cost effective design approach can be developed by using mathematical programming techniques.

The total cost to be minimized is essentially divided into the construction material costs of concrete, steel and formwork. From an economical perspective, it is also desirable to consider in the design process optimization of the critical sections, the nonlinear ultimate behaviour of the concrete and reinforcing steel in accordance with the current design codes (Ferreira, Barros and Barros, 2003). Consideration of serviceability conditions at working loads will be addressed elsewhere as it requires further attention in terms of restrictions on bending moment capacity, stress limitation in the concrete and in steel as a function of cracking conditions, as well as limits on deflections. Such restrictions will have direct consequences on the boundaries of the design space and the feasible design solutions of the optimization problem.

The present paper deals with the structural design optimization of reinforced concrete rectangular sections under ultimate design loads. An analytical approach of the problem, based on a minimum cost design criterion and a reduced number of design variables, is developed. It is shown, among other things, that the problem formulation can be cast into a non linear mathematical programming format. In addition, this study demonstrates the solution of the non linear minimum cost design problem of reinforced concrete rectangular beams under ultimate loads by utilizing a suitable mathematical programming technique. A typical example is presented to illustrate the applicability of the formulation in accordance with the current French design code BAEL-99. The results are then confronted to solutions derived from current design practice.

Problem Formulation

Consider the statically determinate beam of Figure 1, along with the rectangular cross section shown in Figure 2 and let C_0 be the objective function representing the cost of reinforced concrete and reinforcing steel to be used. This function can be defined as:

$$C_0 = L\,[C_c\, b\, d + C_s\, A_s] \qquad (1)$$

where L, b, d are respectively the span, width and effective depth of the beam.
The cost parameters C_0 , C_S and C_c designate the absolute cost and the unit costs of steel and concrete respectively; A_s is the area of reinforcing steel.

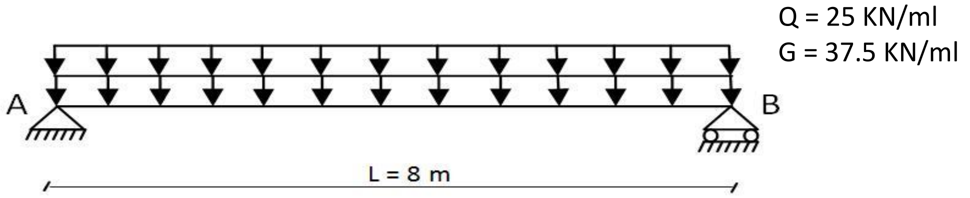

Figure 1. Statically determinate beam with simple supports.

Figure 1. Poutre isostatique sur appuis simple

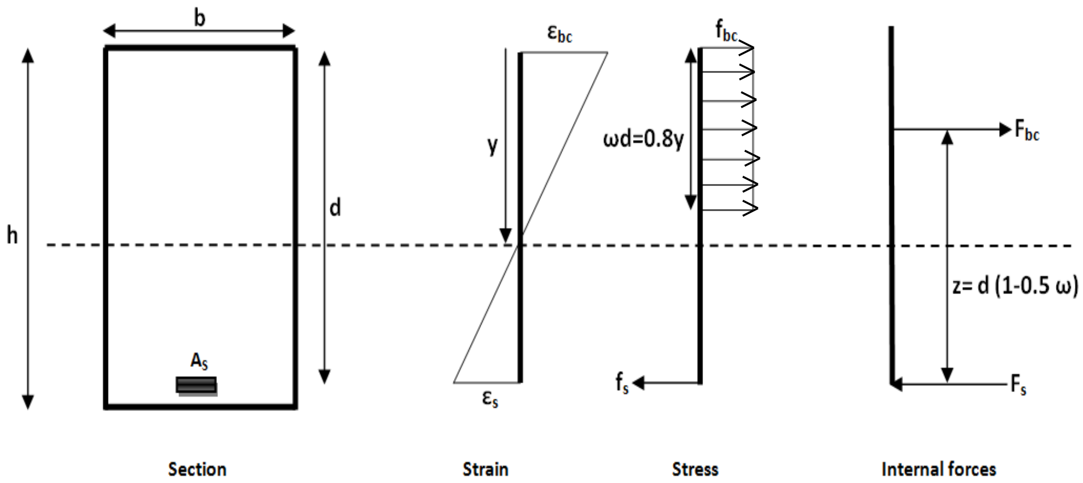

Figure 2. Typical rectangular section, strain, stress and internal force distributions under ultimate design load.

Figure 2. Section rectangulaire type, distribution des déformations, contraintes et efforts internes sous chargement de conception ultime.

The pre-assigned parameters and limitations are as follows:

- beam length: L = 8m
- dead load: G = 37.5 KN/m (including initial self weight)
- live load: Q = 25 KN/m

Combination rule for ultimate beam capacities in bending M_u and shear V_u respectively

- ultimate bending moment capacity: $M_u = 1.35\ M_G + 1.5\ M_Q$
- $M_u = 0.705$MNm
- ultimate shear capacity: $V_u = 1.35\ V_G + 1.5 V_Q$
- $V_u = 3.353$ MN

Input data for concrete characteristics:

- compressive strength of concrete at 28 days: fc28 = 25 MPa
- partial safety factor for concrete: $\gamma_b = 1.5$
- allowable compressive stress: $f_{bc} = (0.85\ f_{c28})/\ \gamma_b = 14.17 MPa$
- allowable shear stress: $\overline{\tau}_u = Min\{(0.20\ f_{c28})/\ \gamma_b\ ; 5MPa\} = 3.25\ MPa$

Input data for steel characteristics:

- elastic limit (steel class HA FeE400): f_e = 400 MPa
- partial safety factor for steel: γ_s =1.15
- allowable tensile stress : $f_s = f_e/\gamma_s$ = 347.8 MPa
- Young's elastic modulus of steel: $E_s = 2x10^5$ MPa
- minimum steel percentage: p_{min} = 0.12%
- maximum steel percentage: p_{max} = 4%

The unit costs ratio of steel and concrete are generally determined in accordance with the book of unit prices for construction materials (Harris, 1999). Cost components such as concrete formwork and steel forming are herein implicitly included in the objective function as appropriate percentages of the unit costs of concrete and steel respectively. The value of the cost ratio C_S / C_c varies from one country to another and may eventually depend from one region to another for certain countries. In the present work, the unit cost ratio C_s / C_c = 36 is used.

It should be equally important to note that in a cost optimization problem, the optimal values of the design variables are affected by the relative cost values of the objective function only, but not by the absolute cost values. Thus, the objective function to be minimized can be written as follows:

$$C = b\,d + (C_S / C_c)\,A_s \qquad Min \longrightarrow \tag{2}$$

The absolute total cost C_0 can then be can recovered from the optimized relative cost C by using the relation $C_0 = C\,L\,C_c$.

In general, the behavior constraints are based on different design codes. For illustrative purposes, the design constraints are herein defined in accordance with the French BAEL-99 design code specifications (Mougin, 2006), a slightly different version of the current EC2 provisions. The case of compressive steel can be included in the formulation without difficulty but for the sake of clarity, it will not be presented in the present illustration.

Thus, the formulation of the minimum cost design of reinforced concrete rectangular beams under ultimate loads can be mathematically stated, without loss of generality, as follows:

Find the decision variables b, h, d, A_s, and ω such that:

$$C = b\,d + (C_S / C_c)A_s \qquad Min \longrightarrow \tag{3}$$

Subject to:

a) Bending stress constraints (see Fig. 2):

$$M_u \le f_{bc}.b.d^2.\omega(1 - 0{,}5\omega) \tag{4}$$

where the relative depth of compressive concrete zone, ω is defined by

$$\omega = (A_s / b.d)\, f_s / f_{bc} = p\, f_s / f_{bc} \tag{5}$$

The ultimate moment capacity of the section in Eq. (4) is based on the assumption that the stress in the concrete under the ultimate design load is uniformly distributed.

To ensure a desired behaviour, the steel percentage (p) is restricted so that

$$p_{min} \le p \le p_{max} \tag{6}$$

$$\omega_{inf} \le \omega \le \omega_{sup} \tag{7}$$

where ω_{inf} and ω_{sup} represent the minimum and maximum relative depths of compressive concrete zone respectively.

b) Conditions on strain compatibility in steel and concrete

$$f_e / \gamma_s E_s \le 0.0035(0.8-\omega)/\omega \le 10/1000 \tag{8}$$

$0.259 \le \omega/0{,}8 \le 0.668$ (steel class HA FeE400) (9)

$0.186 \le \omega(1-0{,}5\omega) \le 0.392$ (steel class HA FeE400) (10)

c) Shear stress constraint:

$$(V_u / \overline{\tau}_u) \leq b\, d \tag{11}$$

d) Design constraints

$$L/15 \leq h \leq L/10 \tag{12}$$

$$d/h = 0.90 \tag{13}$$

$$0.30 \leq b/d \leq 0.50 \tag{14}$$

In this formulation, the amount of steel A_s depends only on the bending stress constraints. All the other constraints are assumed to be functions of b and d only. Thus, for any given concrete dimensions the optimal amount of steel can be found by considering only the bending stress constraints. Furthermore, using Eq.(5), the amount of steel A_s can be expressed in terms of ω, b, d and therefore be eliminated. Since A_s is proportional to ω (for given b and d), the optimal value of ω is the minimal one, satisfying the constraints of Eqs.(4) and (7). This permits the formulation of the optimization problem in the design space of b and d (Fig 3). For any point in the design space, the optimal value of ω is computed directly from the inequalities (4), (7), and the corresponding A_s is then determined by Eq.(5).

The feasible space in Figure 3 can be divided into two regions:

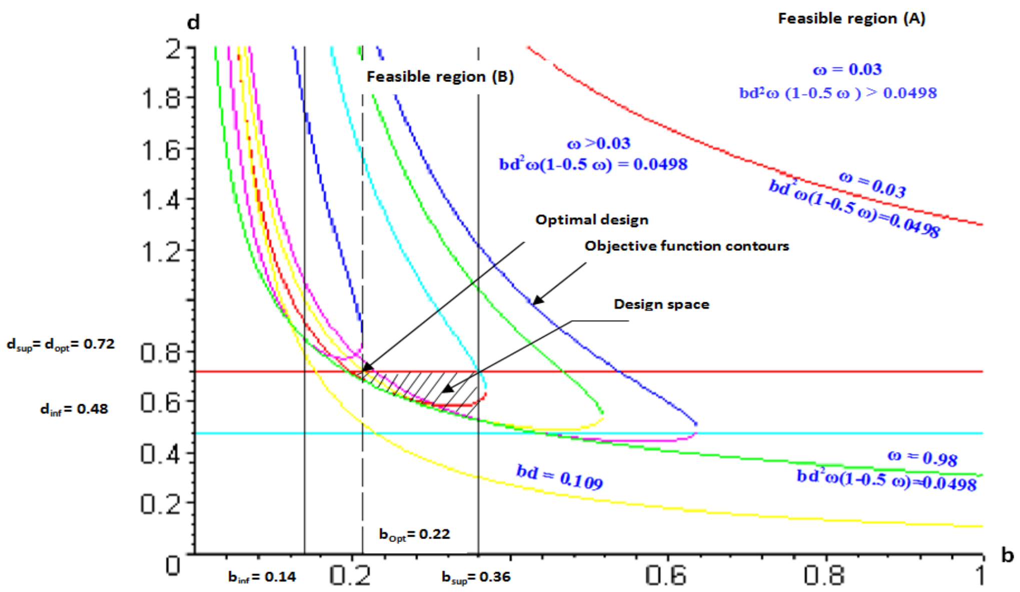

Fig. 3. Design space, reinforced concrete rectangular beam.

Fig. 3. Espace de conception de la poutre rectangulaire en béton armé.

(A) ω = 0.03 and $0.0498 < b.d^2.\omega(1-0.5\omega)$, i.e., the constraint of Eqs.(4) is inactive and ω is determined by the constraint of Eq. (7) (minimum steel percentage).

(B) $0.0498 = b.d^2.\omega(1 - 0.5\omega)$, and $\omega > 0.03$, i.e., the constraint of Eq.(3) is active and the constraint of minimum steel percentage is inactive.

On the boundary between the two regions, both constraints (4) and (7) are active, that is $\omega = 0.03$ and $0.0498 = b.d^2.\omega(1 - 0.5\omega)$. For region (B), we may express the objective function C in terms of b and ω only by eliminating As and d using the equalities (4) and (5).

The objective function equation (3) and the constraints equations, Eq.(4) through Eq.(14), together form a nonlinear optimization problem. The reasons for the non linearity of this optimization problem are essentially due to the expressions for the beam cross sectional area, the bending moment capacity and other constraints equations as well as the requirement to update iteratively the self weight of the element, both in the constraints functions and the objective function. Both the objective function and the constraint functions are non linear in terms of the decision variables (Kirsch, 1993).

In order to solve this non linear optimization problem, the direct search method in a space of two variables representing the concrete cross section dimensions has been used.

Finally, it should also be noted that in the case of serious cracking, the optimization constraints should be reformulated in accordance with the serviceability design loads as prescribed by the design code criteria to be used.

Numerical Results and Discussion

In order to illustrate the applicability of the optimal design obtained by using the present formulation, the minimum cost design procedure is now applied to design the reinforced concrete beam with rectangular section for which a detailed design solution based on the French BAEL design code has been reported by (Thonier, 1982). The pre-assigned parameters and limitations are as given in the preceding section.

It should also be noted that the optimal solution vector of the aforementioned nonlinear mathematical programming problem cannot be considered as the final solution of the minimum cost design problem. As a matter of fact, because of the requirement to update the geometric dimensions of the section with the new self weight of the optimized beam, the degree of nonlinearity of the resulting optimization problem enhances further and the final optimal solution must be obtained through an iterative process (Tiliouine and Fedghouche, 2010).

In the present example, the final optimal solution vector is reached after only 3 cycles of iteration. The corresponding optimal values of the geometric dimensions of the beam are $b_{opt} = 0.22$m and $d_{opt} = 0.72$m in accordance with the graphical solution indicated in the Figure 3. However, for practical considerations the beam width has been set to be equal 0.36m. In this case, the problem is reduced to one-dimensional search in the space of d.

The vector of decision variables as obtained from the optimal design solution using the proposed approach and the classical design solution are shown in Table 1.

Table 1. Final optimal solution including self weight effects.

Solution vector	Classical solution	Practical optimal solution
b(m)	0.50	0.36
d(m)	0.73	0.72
h(m)	0.80	0.80
A_s (m^2)	$32.20x10^{-4}$	$31.70x10^{-4}$
ω	0.2088	0.3002
C	0.48092	0.37334

From the above results, it is clearly seen that the relative depth of the compressive zone associated with the optimal solution is 44% larger than that given by the classical solution, thus leading to a much better use of the concrete. It is also seen that the ratio C_{class}/C_{opt} of relative costs (C_{class} as obtained using BAEL classical design method to the optimal relative cost C_{opt}) is equal to 1.29, i.e. a significant material construction cost saving of the order of 29% as compared to the classical design solution.

A study of the constraints indicated that the design constraints of the beam are all non binding except for the behavior constraints associated with ultimate bending moment capacity (4),the geometrical design constraint (12) and the conditions on strain compatibility (8) and (9). In addition, the results obtained by using various examples have shown that the optimal solutions are insensitive to changes in the shear stress constraints which can thus be excluded from the problem formulation.

Conclusions

This study deals with the minimum cost design of reinforced concrete rectangular beams at ultimate design loads. An analytical approach of the problem based on a minimum cost design criterion and a set of constraints including the nonlinear behaviour of concrete and steel has been formulated.

From the results of the present study, it is possible to draw the following major conclusions:

1. The problem formulation of the optimal cost design of reinforced concrete rectangular-beams can be cast into a non linear programming problem, the numerical solution of which is efficiently determined using a direct search method in a space of only two variables representing the concrete cross section dimensions.
2. Results obtained using various examples show that the optimal solutions are insensitive to changes in the shear stress constraints. Shear stress constraints are usually not critical in the optimal design of reinforced concrete beams and can thus be excluded from the problem formulation.
3. Optimal values of the design variables are affected by the relative cost values of the objective function only, but not by the absolute cost values.
4. The observations of the optimal solutions results reveal that the use of optimization techniques based on the minimum cost design concept may lead to substantial savings in the amount of the construction materials to be used in comparison to classical design solutions of reinforced concrete rectangular-beams.

5. The objective and the constraints considered in the present paper are illustrative in nature. The present approach based on nonlinear mathematical programming can be easily extended to other sections commonly used in structural design. More sophisticated objectives and considerations can be readily accommodated by suitable modifications of the present optimal cost design model.

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